

Math 220 Quiz Solution

February 28, 2008

The Quiz

Find the kernel of the linear map

$$\mathbf{R}^3 \xrightarrow{f} \mathbf{R}^2$$

that is defined for

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \text{ in } \mathbf{R}^3$$

by

$$f(x) = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} .$$

Recommended Solution

1. The kernel of f is the set of all x in \mathbf{R}^3 such that $f(x) = \vec{0}$.
2. Finding this kernel amounts to solving a system of 2 linear equations in 3 variables.
3. Perform elementary row operations on the given matrix so as to maneuver it into reduced row echelon form. (Augmenting it by a zero column would be a waste of time since a zero column remains a zero column under any row operation.)

$$(R_2 \rightarrow R_2 - 2R_1) \quad \begin{pmatrix} 1 & -2 & 0 \\ 0 & 5 & 5 \end{pmatrix}$$

$$(R_2 \rightarrow (1/5)R_2) \quad \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$(R_1 \rightarrow R_1 + 2R_2) \quad \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

4. The resulting system of linear equations is:

$$\begin{cases} x_1 + 2x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = -2x_3 \\ x_2 = -x_3 \end{cases}$$

5. Conclusions:

- The kernel has the parametric form

$$x = t \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} .$$

- The kernel is the linear span (in \mathbf{R}^3) of $(-2, -1, 1)$.