Math 220 Class Slides

February 5, 2008

1 The Assignment

$$f(X) \;=\; f_M(X) \;=\; MX$$

where

$$M \;=\; \begin{pmatrix} 1 & -2 & -1 \\ 5 & 4 & -3 \\ -2 & -3 & 1 \end{pmatrix}$$

2 Task 1: Put M in reduced row echelon form

$$M \;=\; \begin{pmatrix} 1 & -2 & -1 \\ 5 & 4 & -3 \\ -2 & -3 & 1 \end{pmatrix}$$

$$R \;=\; \begin{pmatrix} 1 & 0 & -\frac{5}{7} \\ 0 & 1 & \frac{1}{7} \\ 0 & 0 & 0 \end{pmatrix}$$

3 Task 2: Solve $f(X) = 0$

$$f(X) = MX \qquad M = \begin{pmatrix} 1 & -2 & -1 \\ 5 & 4 & -3 \\ -2 & -3 & 1 \end{pmatrix}$$

Principle: To solve a linear system:

1. Perform row operations on its augmented matrix so that the coefficient matrix portion is placed in reduced row echelon form
2. Apply common sense to the resulting linear system

$$R = \begin{pmatrix} 1 & 0 & -\frac{5}{7} \\ 0 & 1 & \frac{1}{7} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x - \frac{5}{7}z & = & 0 \\ y + \frac{1}{7}z & = & 0 \\ 0 & = & 0 \end{cases}$$

$$\begin{cases} x & = & \frac{5}{7}z \\ y & = & -\frac{1}{7}z \end{cases}$$

z is a parameter.

4 Task 3: What type of geometric object is this?

A line
The line is given in parametric form.

5 Task 4: Characterize Points in the Image of f

The image of f is the set of all points Y with the property that $Y = MX$ for some X.

$$Y = MX = x \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + y \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix} + z \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}$$

- $Y = f(X)$ for some X if and only if Y is a linear combination of the columns of M.
- $Y = f(X)$ for some X if and only if Y is a linear combination of the "pivot" columns of M (in this case the first two columns).

Parametric representation with parameters s and t:

$$Y = s \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$$

The image of f forms a plane through the origin.

6 Task 4: Equation of the Image of f

A plane in $\mathbf{R}^3$ is given by a single linear equation

$$ax + by + cz = 0$$

The vector (a, b, c) of coefficients is a vector that is perpendicular to the plane.
A vector is perpendicular to the image of f if and only if it is perpendicular to each of the pivot columns of M.
Solve

$$\begin{pmatrix} 1 & 5 & -2 \\ -2 & 4 & -3 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$$

and find that (a, b, c) can be taken as any non-zero multiple of $(-1, 1, 2)$.
The equation of the plane is

$$-y_1 + y_2 + 2y_3 = 0$$