Survey of Calculus (Math 106)
Midterm Summary Review

Basic

1. Slope of a line: from any point to another, the change in y divided by the change in x
2. General form of the equation of a line: $ax + by = c$
3. Equation of the line through (a, b) with slope m:
$$\frac{y-b}{x-a} = m$$
4. Equation of the line through (a, b) and (c, d)
$$\text{previous equation with } m = \frac{d-b}{c-a}$$
5. A curve is a graph when it meets vertical lines once
6. The slope of a curve at a point is the slope of the line tangent to the curve at the given point
7. The slope of the graph of f at a point is the value of the derivative f' at the first coordinate of the given point
8. $f'(x) =$ slope of tangent to graph of f at $(x, f(x))$
9. Definition of the derivative as limit of the "difference quotient":
$$f'(x) = \lim_{t\to 0} \frac{f(x+t) - f(x)}{t}$$
10. $f'' =$ derivative of $f' =$ the second derivative of f

Formulas for Derivatives

1. If $f = c =$ constant, then $f' = 0$
2. $(f + g)' = f' + g'$
3. $(f - g)' = f' - g'$
4. $(c_1 f_1 + c_2 f_2 + \ldots + c_n f_n)' = c_1 f_1' + c_2 f_2' + \ldots c_n f_n'$
5. The product rule:
$$(fg)' = fg' + gf'$$
6. The power rule: If $f(x) = x^a$, then $f'(x) = ax^{a-1}$.
7. The quotient rule:
$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$
8. Composition of two functions:
$$(f \circ g)(x) = f(g(x)) \qquad (\text{“}f \text{ following } g\text{”})$$
9. The chain rule (for the derivative of a composition):
   (a) Leibniz notation:
$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

(b) Functional notation:

$$(f \circ g)' = (f' \circ g) \cdot g'$$

(c) Reconciliation:

$$y = f(x) \quad x = g(t) \quad \frac{dy}{dx} = f'(x) = f'(g(t)) \quad \frac{dx}{dt} = g'(t)$$

10. Generalized power rule (application of chain rule with $f(u) = u^a$):

$$\frac{d}{dx} g(x)^a = a g(x)^{a-1} g'(x)$$

11. The exponential rule: If $f(x) = a^x$, then $f'(x) = L(a)a^x$ where

$$L(a) = \lim_{t \to 0} \frac{a^t - 1}{t}$$

(Note: in this it is assumed that the constant base a is positive.)

Exponentials and Logarithms

1. e $(2 < e < 3)$ is the unique number for which $L(e) = 1$, where L is the multiplier appearing in the exponential rule
2. (Important special case of the exponential rule)

$$\frac{d}{dx} e^x = e^x$$

3. Secondary school definition of logarithm:

$$c = \log_a(b) \text{ exactly when } a^c = b \quad (a, b > 0)$$

4. L spawns all logarithms:

$$\log_a(b) = \frac{L(b)}{L(a)} \quad (a, b > 0)$$

5. L is logarithm for the base e or the "natural logarithm":

$$L(a) = \log_e(a) \text{ for each } a > 0$$

6. Derivative of L:

$$L'(x) = \frac{1}{x} \quad (x > 0)$$

7. Derivative of $\log_a$:

$$\frac{d}{dx} \log_a(x) = \frac{1}{L(a)x} \quad (a, x > 0)$$

Graph Sketching

1. Qualitatively accurate sketches may be obtained by plotting only a few points and taking account of information about
   (a) where the function is increasing and decreasing
   (b) where the function is concave up and concave down
   (c) points where the function has local extremes
   (d) points of inflection
   (e) horizontal and vertical asymptotes
2. f is increasing where $f' > 0$, decreasing where $f' < 0$
3. f is concave up where $f'' > 0$, concave down where $f'' < 0$
4. $f'(c) = 0$ if f has a local maximum or minimum when $x = c$
5. $f''(c) = 0$ if the graph of f has an inflection point when $x = c$
6. the line $y = b$ is a horizontal asymptote if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$
7. the line $x = a$ is a vertical asymptote if $f(x)$ becomes infinite (positively or negatively) as $x \to a$