Preliminary Examination in Algebra
August 2009

(1) Determine the number of p-Sylow subgroups in the symmetric group S_p, where p is a prime.

(2) Find all similarity classes of matrices in $M_7(\mathbb{R})$ with the minimal polynomial $(x-1)(x^2+1)^2$. For each class write its rational canonical form.

(3) Show that there is no simple group of order pqr, where $p < q < r$ are prime.

(4) Show that $A \in M_n(k)$, k is a field, is similar to A^T (the transpose of A).

(5) Let $B \in M_n(\mathbb{Q})$ such that $B^5 = 1$ and no eigenvalue of B is equal to 1. Show that n is divisible by 4.

(6) Let F be a field of characteristic zero. Suppose that K/F is finite Galois extension with Galois group G. Prove that if $a \in K$ and $g(a) - a \in F$ for all $g \in G$, then $a \in F$.

(7) Let K be the splitting field over $\mathbb{Q}$, in $\mathbb{C}$, of $x^4 - 2$. Determine the Galois group $Gal(K/\mathbb{Q})$ and the subelds of K. For each subeld F of K, give eld generators over $\mathbb{Q}$.