Math 520A Written Assignment No. 3

due Friday, March 30, 2007

Directions. This assignment should be typeset. You must explain the reasoning underlying your answers. If you make use of a reference other than class notes, you must properly cite its use.

You may not seek help from others on this assignment.

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1. For each field F find a subgroup H of $\mathrm{GL}_n(F)$ such that $\mathrm{GL}_n(F)$ is isomorphic to the semi-direct product of H with $\mathrm{SL}_n(F)$ for the action of the former (by conjugation within $\mathrm{GL}_n(F)$) on the latter.

2. Let F be a field, and let α denote the action by fractional linear transformations of $\mathrm{GL}_2(F)$ on the set $X = F \cup \{\infty\}$.

   (a) Show that α is a transitive action, i.e., there is only one orbit in X.

   (b) What is the isotropy group at ∞?

   (c) Explain briefly why α is induced by an action of $\mathrm{PGL}_2(F)$.

3. Let $\mathbf{F}_4 = \mathbf{Z}/2\mathbf{Z}[t]/(t^2 + t + 1)\mathbf{Z}/2\mathbf{Z}[t]$ be the field with 4 elements. Find the simple quotients for a composition series of $\mathrm{SL}_2(\mathbf{F}_4)$.

4. Find the group of automorphisms of the quaternion group

   $$Q_2 = \left\langle x, y \,|\, x^4 = 1,\ y^2 = x^2,\ yxy^{-1} = x^{-1} \right\rangle \quad .$$

5. One knows that the alternating group A_5 is a simple group of order 60. Prove that any simple group of order 60 must be isomorphic to A_5.