

Math 520A Written Assignment No. 2

due Wednesday, March 2, 2005

Directions. It is intended that you work these as exercises. Although you may refer to books for definitions and standard theorems, searching for solutions to these written exercises either in books or in online references should not be required and is undesirable. If you make use of a reference other than class notes, you must properly cite that use.

You may not seek help from others.

1. For which primes p are the Sylow p -subgroups of the symmetric group S_4 normal?
2. If G is any group, H a subgroup, and x, y elements of G , show that $xH = Hy$ if and only if x and y both belong to the normalizer $N_G(H)$ and determine the same element of $N_G(H)/H$.
3. Show that the group $SL_2(\mathbf{F}_3)$ has a normal subgroup of order 8. List the 8 elements of this subgroup, and explain why this group of order 8 is not isomorphic to the dihedral group D_4 .
4. Let G be a finite group and H a subgroup of index 3 in G that is not a normal subgroup of G . Show that H contains a subgroup N that is normal in G for which $G/N \cong S_3$.
5. Find an explicit list of groups that represent all isomorphism classes of groups of order 66.