Math 502

Written Assignment No. 2

due Tuesday, March 4, 2008

Directions: Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may **not** seek help from others on this written assignment.

Answer the following questions:

1. For each of the following vector triples $[u, v, w]$ and integers $b > 1$ find the rational number r, expressed as the ratio of two integers in base 10 notation, for which u is the sequence of b-adic digits (from left to right) for the integer part of r, v is the sequence of b-adic digits for the portion of the fractional part of r that precedes the repeating portion, and w is the sequence of b-adic digits in the repeating portion of the fractional part of r.

b	u	v	w
10	[5, 1, 8]	[8, 6, 7]	[5, 3, 0, 9]
16	[B, A, D]	[C, 0, D, E]	[F, A, D]
9	[3]	[0, 4]	[2, 0, 0, 8]

2. If $P = a + bx + cx^2 + dx^3 + ex^4 + fx^5$ is a polynomial, find polynomials $q_0, q_1, q_2, \ldots$ of degree at most 1 such that
$$P = \sum_{j \geq 0} q_j (x^2 + 1)^j \quad .$$

3. Factor the polynomial $x^{31} - 1$ as
   (a) a rational polynomial.
   (b) a polynomial mod 2.
   (c) a polynomial mod 3.
   (d) a polynomial mod 5.

4. Find the partial fraction expansion of
$$\frac{1}{x^{11} + 2x^9 + x^7 - x^4 - 2x^2 - 1}$$
   regarded as a "rational function" with:
   (a) rational coefficients.
   (b) coefficients in the integers mod 2.
   (c) coefficients in the integers mod 7.
   (d) coefficients in the integers mod 11.

5. The sequence of integers
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[61, 13, 25, 37, 13, 28, 17, 81, 40, 3, 75, 91, 13, 25, 20, 37, 81,
 25, 91, 95, 25, 91, 17, 70, 37, 17, 25, 77, 25, 75, 85, 25, 13, 91,
 3, 25, 19, 75, 20, 37, 19, 81, 25, 3, 91, 25, 23, 37, 25, 73, 37,
 17, 81, 25, 85, 37, 28, 59, 17, 37, 6]
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   is the sequence of 17-th powers mod 97 of numbers obtained from a sequence of ASCII codes by left-shifting each code 30 units so the sequence occupies the interval $[2, 96]$ instead of the normal printable ASCII interval $[32, 126]$. What text string led to the original sequence?