Introduction to Maple (Math 502)
Assignments

Spring Semester, 2006

Assignments are listed by the **date due**.

PDF and DVI (requires *TeX*[1] software) versions of this page are available for printing. If your browser is prepared to handle MathML, you should view the XHTML version of this page; in the opposite case you should view the classical HTML version.

Most of these assignments are simply exercises designed to prepare you for tests and the written assignments. While you may find it helpful to discuss exercises with others, no collaboration is permitted on the written assignments.

Wed., May. 17:
    Final Examination: 3:30 – 5:30
    The long vector is here[2].

Tue., May. 16:
    Office Hours:

        1:00 – 2:00
        4:30 – 5:30

Mon., May. 15:
    Office Hours:

        3:00 – 5:00

Thu., May. 11:
    Office Hours:

        3:00 – 5:00

Mon., May. 8:
    Last class; **Written Assignment No. 5**[3] (PDF[4] for printing —classical HTML[5] for terminal window browsing) is due.

    There is some updated code for cubic curve arithmetic in the code archive[6]

Wed., May. 3:
    Review Session: Bring questions

Mon., May. 1:

        262: 6, 7, 12 – 14, 16, 17, 20, 21, 24, 25

Wed., Apr. 26:
    Written Assignment No. 4[7] (PDF[8] for printing —classical HTML[9] for terminal window browsing) is due

Mon., Apr. 24:
    Read: §§ 10.5, 10.6

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[1]URI: http://www.tug.org/
[2]URI: vector
[3]URI: ampl060508.xhtml
[4]URI: ampl060508.pdf
[5]URI: ampl060508.html
[6]URI: http://www.albany.edu/~hammond/maple/
[7]URI: ampl060426.xhtml
[8]URI: ampl060426.pdf
[9]URI: ampl060426.html

224: $3 - 5$
260: $3 - 5$

Wed., Apr. 19:
    Read: §§ $10.1 - 10.2$

        260: 1, 2
        And this: Find the area enclosed by the loop of the cubic curve $y^2 = x^3 - x$. Repeat for the cubic curve $y^2 = x - x^3$.

Mon., Apr. 17:
    Read: §§ $9.1 - 9.2$

        224: 1, 2
        And this: Find the polynomial $p(x)$ such that

$$\frac{d^7}{dx^7} \exp(-1/x^2) = \frac{p(x)}{x^{21}} \cdot \exp(-1/x^2) \quad .$$

Mon, Apr 10 — Fri, Apr 14
    University Recess.

Wed., Apr. 5:
    Written Assignment No. 3[10] (PDF[11] for printing —classical HTML[12] for terminal window browsing) is due

Mon., Apr. 3:
    Read: §§ 8.6, 8.7, 8.8

        212: $4 - 6$
        And this: Let p be the prime $128^{15} + 39$. Without trying to solve determine which of the following two congruence equations is solvable:

$$2^m \equiv 11 \pmod{p} \quad \text{and} \quad 11^n \equiv 2 \pmod{p} \quad .$$

    Are you able to solve the solvable one?

Wed., Mar. 29:
    Read: §§ 8.3, 8.4, 8.5

        211: $1 - 3$
        And this: Let p be the prime 1073741827. Find the smallest integers $m, n > 0$ such that $2^m \equiv 3 \pmod{p}$ and $2^n \equiv 13 \pmod{p}$. Repeat this exercise for the modulus $q = 1073741831$ instead of p. What is different with the second modulus?

Mon., Mar. 27:
    Read: §§ 7.5, 7.6, 8.1, 8.2

        188: 5
        211: 1
        And this: Explore the *Maple* function for finding primitive roots mod m, which is `numtheory[primroot]`.

            a. Find the smallest primitive root modulo 289 that is larger than 100.
            b. Find the smallest positive non-prime primitive root mod 40487.
            c. Find the smallest positive number that is primitive modulo both 101 and 103. Is it primitive mod $101 * 103$?
            d. If c is primitive modulo both 101 and 103, what congruence condition on integers $j, k \geq 0$ is equivalent to the condition that $c^j \equiv c^k \pmod{101 * 103}$?

Wed., Mar. 22:
    Read: §§ $7.2 - 7.4$

188: 3, 4

And this: In the context of the last exercise on the previous assignment you have been told that the squeezed vector

```
[71214700618760697933814344423387854991565315314099174321856458 6,
  1786621100356707079804781015651798041041290004401049203827247506,
  1782184643903441535885937756067735301974983951149305281678962346,
  1639000008839632707546680167815675641387259213687418193657940006,
  1535960089185549654706004534787094483505037489361312984436350635,
  1195799297844909964188410557114692983427064185633447219054911622,
  1529236902471918734371483225353942522875473990416411009757742702,
  4099796699996333603474252469274257293697784469965390 51720679885,
  1805600608974887198383474434264987792669166488653256 22675849897,
  1058983644708927766918309320955981103594250701210512127725439642]
```

(where k is maximum, as before, for the given modulus m) may be decrypted with the exponent

$$d \;=\; 6794176380572461023872900844282413489206015741290130394861784 41 \quad.$$

      A. Decrypt it, expand its terms in base 128, and convert the resulting vector, regarded as a sequence of ASCII codes, to a string.

      B. Can you determine what the encrypting exponent was?

Mon., Mar. 20:
    Read: §§ 6.3, 7.1

      174: 5
      188: 1, 2
      And this: Given a vector of digits in base 128 what is the largest block size k for squeezing the vector into a vector of digits for base 128^k so that the resulting squeezed vector can be faithfully encrypted by taking a suitable power of each entry modulo the integer

$$m \;=\; 2468256835981809063232453773840873253369376547681693188080273 739 \ ?$$

Wed., Mar. 15:
    Midterm Test in class

Mon., Mar. 13:
    A light assignment prior to the midterm test:

- Bring review questions.
- Use network resources to find *Maple* code for solving *Sudoku* puzzles, and then find out how long it takes *Maple* to solve this one:

```
..8...1.2
.7.1...94
...3..5..
.8..4.9..
...815...
..1.6..4.
..5..7...
79...6.1.
6.4...2..
```

WARNING: Be careful when downloading code from the network. First make sure a location where you find code is trustworthy, and then look over any code before running it.

Wed., Mar. 8:

    Written Assignment No. 2[13] (PDF[14] for printing —classical HTML[15] for terminal window browsing) is due.

    Examples of procedures for Monday's assignment are now in the code archive[16].

Mon., Mar. 6:

    Read: §§ 6.1 – 6.2

        174: 2

        And this, which is related to cryptography:

    Write *Maple* procedures that operate on lists of integers as follows:

1. Takes two arguments, a list name v and an integer a, and returns a new list in which each entry x_j of the input list is replaced by $x_j + a$.

2. Takes three arguments, (a) a list of digits relative to a base b (the first argument), (b) the base $b > 1$ (second argument), and (c) an integer $k \geq 1$, and returns a list of digits relative to b^k as base in which the successive entries of the new list are the integers in the interval $\left[0, b^k - 1\right]$ that are represented in base b by the successive sequences of length k in the input list. In each expansion the first digit will be "most significant" and the input sequence should be internally "padded", i.e., padded inside the procedure, with trailing zeros, if necessary, in order to make its total length a multiple of k.

3. Takes three arguments, (a) a list of digits relative to base b^k, (b) the integer $b > 1$, and (c) the integer exponent $k > 0$, and returns a sequence of digits relative to base b obtained successively in groups of length k by expanding each input integer in base b, with the "most significant" digit listed first and any trailing 0's arising from expansion of the last input integer removed.

    Note that, for given values of the second and third arguments, the last two procedures invert each other.

Wed., Mar. 1:

    Read: §§ 5.3 – 5.4

        151: 3 – 5

        And this: Conduct some experiments in cryptography using computers[17] (PDF[18] for printing —classical HTML[19] for terminal window browsing).

Mon., Feb. 27:

    Announcement: The midterm test will be held on Wednesday, March 15.

    Read: §§ 5.1 – 5.2

        151: 1, 2

        And this: Write a Maple procedure that given a univariate polynomial $f(x)$ and a polynomial $b(x)$ of degree at least 1 returns the vector of coefficients $c_j(x)$ for the b-adic expansion of $f(x)$

$$f(x) \;=\; \sum_{j \geq 0} c_j(x) b(x)^j$$

    where $\deg(c_j(x)) < \deg(b(x))$ for each $j \geq 0$.

Mon. − Fri., Feb. 20 − 24 University Winter Recess

Wed., Feb. 15:

    Scan: Chapter 4

    Exercises:

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[13]URI: ampl060308.xhtml
[14]URI: ampl060308.pdf
[15]URI: ampl060308.html
[16]URI: http://www.albany.edu/~hammond/maple/
[17]URI: cbc.xhtml
[18]URI: cbc.pdf
[19]URI: cbc.html

137: 1, 4

And this: Write a Maple procedure that given a base $b \geq 2$ and a triple of vectors equivalent to the base b representation of a positive rational number — each vector consisting of digits relative to the base b, with the vectors in order being (a) the digit sequence (possibly empty) to the left of the decimal point, (b) the digit sequence (possibly empty) to the right of the decimal point before the repetition pattern, and (c) the digit sequence (if any) that repeats — returns the positive rational number as a fraction m/n where m and n are positive integers without common divisor.

Mon., Feb. 13:

Written Assignment No. 1[20] (PDF[21] for printing —classical HTML[22] for terminal window browsing) is due

Wed., Feb. 8:

Read: §§ 3.4 – 3.6

Exercises:

93: 6 – 10

Find: the length of the repeating pattern in the (base 10) decimal expansion of $355/113$ (which is the 4-th convergent in the continued fraction expansion of π). Repeat this exercise for the base 2 expansion of the same rational number.

And this: Write a Maple procedure that when given a finite continued fraction, presented as the vector $[a_0, a_1, a_2, a_3, \ldots, a_n]$ representing

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ldots + a_n}}} \; ,$$

with the a_i all integers and $a_i \geq 1$ for $i \geq 1$, returns the rational number it represents.

Previous assignment: procedure[23] written for the last exercise.

Mon., Feb. 6:

Read: §§ 3.1 – 3.3

Exercises:

63: 12, 13
93: 1 – 5

And this: Write a *Maple* procedure that for two given integers $m, n > 0$ returns a vector of vectors $[v, w]$ where v is the vector of successive quotients and w the vector of successive remainders in application of the Euclidean algorithm to the pair (m, n).

More *Maple* code: See this[24].

Wed., Feb. 1:

Read: §§ 2.5 – 2.6

Exercises:

63: 6 – 11

And this: Examine all iterates of the Syracuse function applied to each integer n up to $10,000$ and find the integer n in that range having an iterate $s_k(n)$ for which the ratio $s_k(n)/n$ of the iterate to the starting integer is largest. *Hint:* If the problem is modified to consider only integers n up to 100, then the integer in that smaller range having an iterate with largest ratio is 27, and the iterate presenting the largest ratio is $s_{77}(27) = 9232$.

About the Syracuse function: It is generally conjectured that for each integer $n > 1$ there is some integer $k > 1$ such that the kth iterate of s applied to n is 1. No proof of this has been known. For more information see the survey article[25] by J. C. Lagarias found at *arXiv*[26].

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[20] URI: ampl060213.xhtml
[21] URI: ampl060213.pdf
[22] URI: ampl060213.html
[23] URI: http://www.albany.edu/~hammond/maple/mcf
[24] URI: http://www.albany.edu/~hammond/maple/
[25] URI: http://www.arxiv.org/pdf/math.NT/0309224
[26] URI: http://www.arxiv.org/

***Maple* code for the Syracuse function:** This is essentially the same as what was shown in class on Wednesday the 25th. It is provided for those who may still be having trouble getting it to run.

```
syr := n -> if n <= 1 then 1 elif n mod 2 = 0 then n/2 else 3*n+1 fi;
```

Mon., Jan. 30:
Read: §§ 2.1 – 2.4
Exercises:

63: 1 – 5
And this: The Syracuse function s is defined for integers n by

$$s(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 3n+1 & \text{if } n > 1 \text{ is odd} \\ n/2 & \text{if } n > 1 \text{ is even} \end{cases}$$

The *iterates* of s are

$$s_1(n) = s(n), \ s_2(n) = s(s(n)), \ s_3(n) = s(s(s(n))), \ldots \quad .$$

For example, $s_1(6) = s(6) = 3$, $s_2(6) = s(3) = 10$, $s_3(6) = s(10) = 5$, $s_4(6) = s(5) = 16$, $s_5(6) = s(16) = 8$, $s_6(6) = s(8) = 4$, $s_7(6) = s(4) = 2$, $s_8(6) = s(2) = 1$. Since the 8th iterate of s applied to 6 is 1, all higher iterates of s applied to 6 are 1.

Find the 5 smallest values of n for which the first $2n+1$ iterations of s applied to n fail to yield 1.

Wed., Jan. 25:
Acquire the textbook. Read through chapter 1, and try some of what is sketched there for yourself in *Maple*.

About free *general purpose* computer algebra systems: The following items were found through a web search, but none of them have been reviewed.

Axiom [27]
Axiom has been in development since 1973 and was sold as a commercial product. It has been released as free software under the Modified BSD License. It is sponsored by CAISS, the Center for Algorithms and Interactive Scientific Software, at The City College of New York.

Maxima [28]
Maxima is a descendant of *Macsyma*, the computer algebra system developed in the late 1960s at the Massachusetts Institute of Technology. It is free under the GNU General Public License subject to some export restrictions from the U.S. Department of Energy. A proprietary version of *Macsyma* is also available.

Mon., Jan. 23:
First meeting: No assignment.

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[27]URI: http://savannah.nongnu.org/projects/axiom
[28]URI: http://maxima.sourceforge.net/