Math 502

Written Assignment No. 3

due Wednesday, April 5, 2006

Directions: Use *Maple* for assistance in responding to the following problems. Please typeset your solutions. Explain what you have done. *Maple* session details are not necessary unless you think it important to include them. Accuracy is important.

Although you may refer to books and notes, you may not seek help from others on this written assignment.

1. Determine polynomials $f_0(x), f_1(x), f_2(x), \ldots$ such that

$$(xy^3 + 1 + x + x^2y)^3 \;\; = \;\; \sum_{k \geq 0} f_k(x)y^k \quad .$$

2. Find the least non-negative integer x that solves the congruence equation

$$10^x \equiv 2 \pmod{40487^7} \quad .$$

3. How can the length 15 string represented by the vector of ASCII codes

    `[97, 39, 98, 96, 99, 34, 100, 10, 101, 39, 102, 34, 103, 96, 104]`

   — which contains a newline and various quoting characters — be entered as an input string in Maple without reference to the ASCII codes?

4. The vector

    `[13741233678124922402948194114510172400702323945729017670168638 46,`
    `83931650248492846360048282081073837682172525043232025317434618 2,`
    `12397383275966782083409856333383872278585263173466036407917733 35,`
    `17861404053735357370868954408532307827729012825814144885213991 67,`
    `50066543278884354314529521092114258862260087631491800520914255 0,`
    `11347917245669685924210207508731611585816573880170518548676998 47,`
    `22890425760822706120593530482403491986040283645642251502150427 59]`

   represents a sequence of ASCII codes (range 0 to 127) that has been squeezed into blocks of 30 and then encrypted by taking a certain power modulo

    $m \;\; = \;\; 24682568359818090632324537778249064069954668657168038653014017 33 \quad .$

   It may be decrypted by using the exponent 5^{15}. What string was encrypted?

5. Write code for a Maple procedure that takes an **expression** e in two variables x, y as first argument and the variable names as second and third arguments and returns the anonymous **function** of two arguments

$$(x, y) \longmapsto x\frac{\partial e}{\partial x} + y\frac{\partial e}{\partial y} \quad .$$

   Make sure that the code you submit works correctly.