Transformation Geometry — Math 331

February 11, 2004

Four Kinds of Isometries of the Cartesian plane

- Recall that by definition an *isometry* of $\mathbf{R}^n$ is a distance-preserving affine transformation. Please refer to the discussion accompanying the assignment due February 4 as well as to exercises involving isometries of $\mathbf{R}^2$ posed in recent assignments.
- Four types of isometries of $\mathbf{R}^2$ are (a) rotations, (b) translations, (c) reflections, and (d) glide reflections.
- A rotation has a point called its center. Under a rotation every point is moved through a fixed angle around the circle with the given center on which it lies. A rotation is orientation-preserving.
- A *translation* is an affine transformation of the form $x \mapsto x + v$. A translation is orientation-preserving.
- A reflection has a line called its axis. Under a reflection every point is sent to its mirror image relative to the axis. The line segment from a point to its image under the reflection is perpendicularly bisected by the axis. A reflection is orientation-reversing.
- A *glide reflection* is the transformation that results when a reflection is followed with the translation by a non-zero vector parallel to the axis of the reflection. A glide reflection is an orientation-reversing isometry with no fixed point.
- The identity transformation may be regarded as both a translation and a rotation, but is sometimes regarded as neither.

Exercises due Friday, February 13

Let f be the isometry of $\mathbf{R}^2$ that is obtained by following rotation about the origin counter-clockwise through the angle $\pi/6$ with translation by the vector $(2, 0)$.

1. Find a 2×2 matrix U and a vector v in the plane such that $f(x) = Ux + v$ for each x in $\mathbf{R}^2$.
2. Find a point c in $\mathbf{R}^2$ such that $f(c) = c$.
3. It being claimed without proof for the moment that every isometry of $\mathbf{R}^2$ falls into one of the four classes enumerated above, explain from that why this f must be a rotation.
4. Give a geometric construction of the center of rotation for f.
5. What is the angle of rotation for f?