Transformation Geometry — Math 331

January 26, 2004
revised February 1, 2004

Comment on exercises

Let A, B, and C be the points in the Cartesian plane that are given by

$$A = (0,-1)\ ,\quad B = (3,4)\ ,\quad \text{and}\quad C = (-1,1)\quad .$$

- The point where the **perpendicular bisectors** of ΔABC meet is the point $uA+vB+wC = (43/22, 27/22)$ where

  $$u = \frac{175}{242}\ ,\quad v = \frac{135}{242}\ ,\quad \text{and}\quad w = -\frac{34}{121}\quad .$$

  This coefficient vector is the unique vector (u,v,w) with $u+v+w = 1$ that is parallel to the vector $(\sin\alpha\cos\alpha, \sin\beta\cos\beta, \sin\gamma\cos\gamma)$ where α, β, and γ are the vertex angles in ΔABC. The common distance of the intersection point from the three vertices is $\frac{5\sqrt{170}}{22}$.

- The point where the **angle bisectors** of ΔABC meet is the point $uA+vB+wC$ where

  $$(u,v,w) = \left(\frac{a}{a+b+c}, \frac{b}{a+b+c}, \frac{c}{a+b+c}\right)$$

  with a, b, and c the lengths of the sides of ΔABC, which, respectively, are 5, $\sqrt{5}$, and $\sqrt{34}$. Therefore,

  $$uA+vB+wC = \left(\frac{3\sqrt{5}-\sqrt{34}}{5+\sqrt{5}+\sqrt{34}}, \frac{-5+4\sqrt{5}+\sqrt{34}}{5+\sqrt{5}+\sqrt{34}}\right)\quad .$$

  Note that by the law of sines this coefficient vector, which is parallel to the vector of lengths of sides is also parallel to the vector $(\sin\alpha, \sin\beta, \sin\gamma)$. The common distance of the intersection point from the three sides is $\frac{11}{5+\sqrt{5}+\sqrt{34}}$.

Exercises due Wednesday, January 28

The first two of the following exercises will be important for later work.

1. Prove the *fulcrum principle*: If A and B are different points, l the length of the segment AB, and $P = uA+vB$ with $u+v = 1$, then the length of the segment AP is $|v|\,l$ and the length of BP is $|u|\,l$.

2. Prove the principle of *preservation of proportionality in barycentric coordinates*: If A, B, and C are three non-collinear points and $P = uA+vB+wC$ with $u+v+w = 1$, then the line AP meets the line BC at the point $(1/(v+w))(vB+wC)$ provided that $v+w \neq 0$ or, equivalently, $P \neq A$ and the line AP is not parallel to the line BC.

3. Let A, B, and C be three non-collinear points, D a point of the line segment AC, and E a point of the line segment BC. Use barycentric coordinates relative to A, B, and C to show that the line **segments** AE and BD meet.