Transformation Geometry — Math 331

January 21, 2004

Discussion

- A **linear combination** of points or vectors $v_1, \ldots, v_N$ is any point of the form
$$\sum_j c_j v_j$$
where the c_j are numbers. The numbers c_j are called the *coefficients* of the linear combination of the given points or vectors.
- **Definition:** The sum
$$\sum_j c_j$$
of the coefficients in a linear combination is called the *weight* of the linear combination.
- A **barycentric combination** of points or vectors $v_1, \ldots, v_N$ is any weight 1 linear combination of them in which each coefficient is non-negative.
- If A and B are two different points of the plane (or of space or of n-dimensional space), then the line determined by A and B is the set of all weight 1 linear combinations of A and B, and the line segment between A and B is the set of all barycentric combinations of A and B. Note that if $V = B - A$ is the vector from A to B, then the line determined by A and B is the set of all points $A + tV$, and the line segment AB is the subset of these points with $0 \leq t \leq 1$.
- **Theorem.** If A, B, and C are any non-collinear points in the Cartesian plane, then every point X of the plane is a unique weight 1 combination of A, B, and C.
- If A, B, and C are any non-collinear points in the Cartesian plane, then a point X in the plane lies in the triangle determined by the three points if and only if it is a barycentric combination of A, B, and C.

Exercises due Friday, January 23

Let A, B, C, and D be the points in the Cartesian plane that are given by
$$A = (0, -1)\ , \quad B = (3, 4)\ , \quad C = (-1, 1)\ , \quad \text{and} \quad D = (1, 2)\ ,$$
and let T be the triangle with vertices A, B, and C.

1. Find the midpoint of the line segment AB.
2. For which values of t does the point $(1 - t)A + tB$ lie on the line segment AB?
3. Find the point where the line AC meets the line BD. Does this intersection point lie on both of the line segments AC and BD?
4. Find the point where the three medians of T meet.
5. Find the point where the three perpendicular bissectors of the sides of T meet.
6. Find the barycentric coordinates of the point $(2, 2)$ with respect to the vertices of the triangle T.
7. How much information about the topic of *barycentric coordinates* can you find on the world wide web?

Document network location for HTML:
`http://math.albany.edu:8000/math/pers/hammond/course/mat331/assgt/tg040123.html`