# Written Assignment No. 1

### due September 28, 2005

**General Directions:** Written assignments should be submitted typeset. What you submit must represent your own work.

## Example of a Solved Exercise

Please note that the directions for this solved exercise differ from those for the exercises in the present assignment.

*Prove the following statement:* If the number of elements in a finite group $G$ with identity $e$ is even, show that there is at least one element $g$ in $G$ such that $g \neq e$ but $g * g = e$.

*Proof.* Let $2n$ be the number of elements of the given finite group $G$. The assertion is that there is at least one element of $G$ other than $e$ for which $g * g = e$, i.e., $g = g^{-1}$. If this were not the case then for every $g \neq e$ in $G$ one would have $g \neq g^{-1}$, i.e., $g$ and $g^{-1}$ would be different elements. So the set $G - \{e\}$ would be the disjoint union of two element subsets of the form $\{g, g^{-1}\}$, and, therefore, the number $|G - \{e\}|$ of elements of $G - \{e\}$ would be even. Since $G$ is the disjoint union of $\{e\}$ and $G - \{e\}$,

$$|G| \; = \; 1 + |G - \{e\}| \ ,$$

and, therefore, the number of elements of $G$ would be odd. Hence, if the number of elements of $G$ is even, there must be at least one element of $G\{e\}$ for which $g * g = e$.

## Assigned Exercises

**Read these directions carefully:** for each of the following statements either provide a proof that the statement is true or label the statement as false and provide justification.

1. The multiplicative group of the integers mod 11 is a cyclic group.
2. If $G$ is an abelian group with identity $e$, then the set $T$ of all elements $t \in G$ such that $t^2 = e$ is a subgroup of $G$.