

Classical Algebra

Written Assignment No. 2

due Friday, October 5, 2007

Directions

Written assignments must be typeset.

While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing, give major steps in computation, and explain any reasoning used.

Accuracy is important. With 5 problems in an assignment worth 10 points, there is limited room for partial credit on a problem.

Problems

1. Find (i) the least non-negative residue and (ii) the residue of least absolute value for 2^{213} modulo 1025.
2. Find all points (x, y) with integer coordinates on the line

$$129x + 111y = 33 \quad .$$

3. List *all* solutions that are distinct mod 40 for each of the following congruences:
 - (a) $3x \equiv 1 \pmod{40}$.
 - (b) $3x \equiv 16 \pmod{40}$.
 - (c) $28x \equiv 41 \pmod{40}$.
 - (d) $47x \equiv 21 \pmod{40}$.
 - (e) $25x \equiv 55 \pmod{40}$.
4. List the *number* of distinct solutions mod 283409 for each of the following congruences:
 - (a) $42x \equiv 791 \pmod{283409}$
 - (b) $42x \equiv 256 \pmod{283409}$
 - (c) $42x \equiv 14 \pmod{283409}$
5. Prove that a and b have least common multiple ab if there exist integers c and d such that

$$ac + bd = 1 \quad .$$