Classical Algebra

Written Assignment No. 2

due Friday, October 10, 2003

Directions: Written assignments must be typeset. While it is neither necessary nor desirable to show small details of computation, you must indicate what you are doing and explain any reasoning used. Accuracy is important in completing this assignment.

If you are in the writing intensive division of the course, you must complete each written assignment in a satisfactory way. This may require re-submission, possibly more than once, after the initial evaluation.

1. Find the least non-negative residue of $2^{129} \pmod{1025}$.

2. List *all* solutions that are distinct mod 40 for each of the following congruences:

   (a) $3x \equiv 1 \pmod{40}$.

   (b) $3x \equiv 25 \pmod{40}$.

   (c) $28x \equiv 43 \pmod{40}$.

   (d) $59x \equiv 74 \pmod{40}$.

   (e) $25x \equiv 55 \pmod{40}$.

3. List the *number* of distinct solutions mod 121461 for each of the following congruences:

   (a) $48x \equiv 771 \pmod{121461}$

   (b) $48x \equiv 6 \pmod{121461}$

   (c) $48x \equiv 256 \pmod{121461}$

4. Prove that a and d have no common factor if there exist integers b and c such that

$$ac + bd \; = \; 1 \quad .$$

5. Let a, b, and c be integers with $a, b > 0$. Prove that if there is an integer point (r, s) on the line

$$ax + by \; = \; c \; ,$$

   then there is one and only one integer point (x, y) on the line for which

$$0 \leq x < \frac{b}{\gcd(a, b)} \quad .$$