Classical Algebra (Math 326)
Written Assignment

due Monday, Dec 9, 2002

Directions

There will be a premium placed on accuracy in the grading of this assignment. Please submit your assignment typed. If there is more than one page, please staple.
Explain your solutions.

Problems

1. Find the smallest positive primitive root modulo each of the following primes:
   (a) 23.
   (b) 43.
   (c) 71.

2. Find the order of the congruence class of the polynomial $f(x)$ modulo the polynomial $m(x)$ when the field of coefficents is $\mathbf{F}_p$ in the following cases:
   (a) $f(x) = x$, $m(x) = x^2 + 1$, and $p = 5$.
   (b) $f(x) = x$, $m(x) = x^2 - x + 1$, and $p = 5$.
   (c) $f(x) = x - 2$, $m(x) = x^2 + 5x + 1$, and $p = 7$.
   (d) $f(x) = x + 1$, $m(x) = x^3 - x^2 + 1$, and $p = 3$.

3. Find a polynomial $f(t)$ in $\mathbf{F}_5[t]$ whose congruence class modulo $m(t)$ is a primitive element for the field $\mathbf{F}_5[t]/m(t)\mathbf{F}_5[t]$ when $m(t) = t^2 - t + 1$.

4. $\mathbf{F}_4$ is defined to be the field $\mathbf{F}_2[t]/(t^2 + t + 1)\mathbf{F}_2[t]$.
   (a) How many congruence classes are there of polynomials in $\mathbf{F}_4[x]$ modulo the polynomial $x^3 + x + 1$?
   (b) Find a primitive element for the ring $\mathbf{F}_4[x]/(x^3 + x + 1)\mathbf{F}_4[x]$ of congruence classes.
   (c) Explain why the polynomial $x^3 + x + 1$ is irreducible over $\mathbf{F}_4$.

5. Write a proof of the following proposition: If F is a field and $f(x)$ is in the ring $F[x]$ of polynomials with coefficients in F, then the polynomial x and the polynomial $f(x)$ have no (non-constant) common factor if and only if $f(0) \neq 0$.