

# Linear Algebra (Math 220)

## Assignment due Thursday, April 24

### 1 Preparation

Expect a **quiz**.

#### Relevant Reading:

Lay §§ 5.1 – 5.3

Hefferon §§ 5.I – 5.II

### 2 Exercises

1. When  $\mathbf{h}$  is the basis of the Cartesian plane with  $h_1 = (a, b)$  and  $h_2 = (c, d)$ , what is the matrix of the rotation about the origin through the angle  $\pi/2$  relative to  $\mathbf{h}$ ? (Assume that  $ad - bc \neq 0$ .)
2. Let  $f$  be the linear function from  $\mathbf{R}^3$  to  $\mathbf{R}^3$  that has the matrix

$$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

relative to the basis of  $\mathbf{R}^3$  given by the columns of the matrix

$$\begin{pmatrix} 3 & 6 & 2 \\ 2 & -3 & 6 \\ 6 & -2 & -3 \end{pmatrix} .$$

- (a) How many lines  $L$  passing through the origin have the property that  $f$  carries each point of  $L$  to a point of  $L$ ?
  - (b) Find all points  $x$  in  $\mathbf{R}^3$  for which  $f(x) = x$ .
  - (c) For each of two different lines through the origin find a point on the line that is carried to another point on the same line.
3. Let  $S$  be the  $2 \times 2$  matrix

$$\begin{pmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{pmatrix} .$$

- (a) Find a point  $P$  in  $\mathbf{R}^2$  at distance 1 from the origin for which  $SP = -P$ .
- (b) Find a line in  $\mathbf{R}^2$  characterized by the property that the matrix  $S$  represents the reflection in that line relative to the standard basis of  $\mathbf{R}^2$ .
- (c) Find an *orthogonal* matrix  $U$  for which

$$U^{-1}SU$$

is a diagonal matrix.